

INTERNATIONAL UNIVERSITY

MID-TERM TEST

November 2005

CALCULUS I

(For Classes IT, ET, and BA)

Lecturers: Assoc. Prof., Dr. Nguyen Dinh

Dr. Janet Caroline Harris

Time allowed: 90 minutes

Document prohibited (except dictionaries).

Answer any FOUR of the following six questions.

Each question carries a total of 25 marks.

QUESTION 1. There are two common systems for measuring temperature: Celsius and Fahrenheit. Water freezes at 0°C and 32°F ; it boils at 100°C and 212°F .

(a) Assuming the Celsius temperature T_C and the Fahrenheit temperature T_F are related by a linear equation, find this equation.

(b) What is the slope of the line relating T_F and T_C if T_F is plotted on the horizontal axis?

(c) Normal body temperature is 98.6°F . What is it in $^{\circ}\text{C}$?

QUESTION 2. Let

$$f(x) := \begin{cases} -x & \text{if } x \leq -1, \\ 1 - x^2 & \text{if } -1 < x < 1, \\ x - 1 & \text{if } x > 1. \end{cases}$$

(a) Evaluate each of the following limits, if it exists.

$$(i) \lim_{x \rightarrow 1^+} f(x); \quad (ii) \lim_{x \rightarrow 1} f(x); \quad (iii) \lim_{x \rightarrow -1^+} f(x);$$

$$(iv) \lim_{x \rightarrow -1^-} f(x); \quad (v) \lim_{x \rightarrow 0} f(x).$$

(b) At which points in its domain, the function is continuous, discontinuous? Give a brief explanation for your conclusions.

(c) Sketch the graph of f .

QUESTION 3. (a) Let $f(x) = x^5 - 2x^4 + 2$, and $g(x) = x^4 + 3x^3 - x - 1$. Show that the equation $f(x) = 0$ has at least one solution and the equation $g(x) = 0$ has at least two solutions.

(b) Show that the conclusions in (a) are still valid if $f(x)$ is a polynomial of odd order and $g(x)$ is a polynomial of even order such that there exists $\bar{x} \in \mathbb{R}$ with $g(\bar{x}) < 0$.

QUESTION 4. Suppose that f is a differentiable function with the property that $f(x + y) = f(x) + f(y) + 5xy$ for all $x, y \in \mathbb{R}$ and $\lim_{h \rightarrow 0} \frac{f(h)}{h} = 3$. Find $f(0)$ and $f'(x)$, for all $x \in \mathbb{R}$.

QUESTION 5. A certain cell culture grows at a rate proportional to the number of cells present. If the culture contains 500 cells initially and 800 after 24 hours, how many cells will be present after a further 12 hours?

QUESTION 6. Let

$$f(x) := \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(a) Determine whether or not $f'(0)$ exists.

(b) Find $f'(x)$ (where it exists).